Drive Power Considerations for Externally Loaded Resonant Cavities

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Abstract: *The drive power equation for a resonant cavity with two coupled ports is reviewed. Some fundamental properties of this equation on resonance are discussed. In particular when one desires a loaded quality factor much lower than the unloaded quality factor of the cavity, the drive power is minimized when one chooses to overcouple the drive port to the cavity.*

Introduction

The forward drive power required to maintain a certain amount of steady-state stored energy on resonance within a cavity with two externally coupled ports is given as

$$P_{FWD} = \frac{\left(1+\beta_1+\beta_2\right)^2}{4\beta_1} \cdot \frac{\omega_o U_{cav}}{Q_o} \quad , \quad (1)$$

where P_{FWD} is the power associated with the forward wave on the input transmission line of the driven port 1, β_1 and β_2 are the coupling coefficients of the coupler / cavity interface at ports 1 and 2 respectively, ω_o is the radian resonant frequency, U_{cav} is the energy stored in the cavity in steady state, and Q_o is the unloaded quality factor of the cavity. The derivation of this equation is explained in Appendix A.

The well-known facts that the forward power decreases with increasing Q_o and that the forward power is minimized by choosing $\beta_1 = 1$ and $\beta_2 = 0$ are seen from Eq. 1 and its first and second derivatives with respect to β_1. With $\beta_1 = 1$ the loaded quality factor, Q_L, would be equal to $\frac{1}{2}Q_o$.

The above statements may seem trivial, but what if it is impractical to operate with $Q_L = \frac{1}{2}Q_o$? This situation arises with superconducting cavities whose Q_o values can reach the 10^{10} range. A high Q_o may minimize cavity losses but becomes a nuisance when one considers perturbations to the resonant frequency caused by microphonics and Lorentz-force detuning. At a resonant frequency of 4GHz, the loaded 3dB bandwidth of a unity coupled cavity with $Q_o = 1 \cdot 10^{10}$ is only 0.8 Hz. To control the cavity energy and amplitude in the presence of disturbances with such a low bandwidth is a difficult task. Thus, one considers operating at lower values of Q_L.

Lowering Q_L

The loaded quality factor for a cavity with two coupled ports can be expressed[1] as

$$Q_L = \frac{Q_o}{(1+\beta_1+\beta_2)} \quad . \quad (2)$$

Q_L can be lowered by decreasing Q_o, increasing β_1, increasing β_2, or any combination of these three. Regardless of how a lower Q_L is achieved, the following expression results from substituting Eq. 2 into Eq. 1,

$$P_{FWD} = \frac{\left(1+\beta_1+\beta_2\right)}{4\beta_1} \cdot \frac{\omega_o U_{cav}}{Q_L} \qquad (3)$$

If one is interested in minimizing P_{FWD} for a particular Q_L, Eq. 3 recommends making $\beta_2 = 0$ and making β_1 as large as possible, while still satisfying Eq. 2 of course. When one desires $Q_L << Q_o$ with $\beta_2 = 0$, by Eq. 2 $\beta_1 >> 1$. Thus P_{FWD} approaches its asymptotic value of $\frac{1}{4}\frac{\omega_o U_{cav}}{Q_L}$.

The choice of lowering Q_L by decreasing Q_o does not minimize P_{FWD}. From Eq. 2 any decrease in Q_o will necessitate a decrease in β_1 to achieve the same Q_L. The first derivative of Eq. 3 says that $\frac{\partial P_{FWD}}{\partial \beta_1} < 0$. Any decrease in β_1 would only increase P_{FWD} for the same Q_L.

Thus, when one wishes to operate with a Q_L that is lower than ½ of an achievable Q_o, the required forward drive power is minimized by choosing to overcouple the drive port rather than either loading the cavity with a second port or decreasing Q_o with a more lossy cavity material.

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[1] see C.G. Montgomery, R.H. Dicke, E.M. Purcell, "Principles of Microwave Circuits", McGraw-Hill Book Co., 1948, Chapter 7.

Conclusion

It has been shown that the required forward power to drive a resonant cavity that has an unloaded Q much higher than the desired operating loaded Q is minimized by choosing to overcouple the input port. This is commonly seen in superconducting practice and may not be anything new to many. However it is seldom explained that this is the optimal way (in terms of drive power requirements) of achieving a practical loaded Q. This practice can be applied to any resonant cavity. It is not restricted to the superconducting case. The overcoupled case takes advantage of the increase in the voltage at the end of the transmission line caused by the reflected wave. Of course to protect the source one needs to use a circulator. This certainly is not impractical especially at higher frequencies where dimensions of circulators are small.

Appendix A – Derivation of the Drive Power Requirement Equations

An equivalent circuit model[1] representing a cavity with two coupled ports is shown in Figure 1. Port 1 with characteristic impedance Z_{o1} is coupled to the cavity through a transformer with turns ratio N1. Port 2 with characteristic impedance Z_{o2} is coupled to the cavity through a transformer with turns ratio N2. The cavity resonant mode of interest is modeled with a parallel RLC circuit. The system is driven by a source located on port 1.

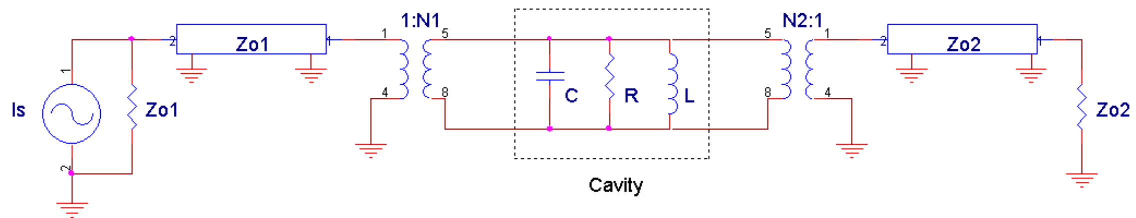

Figure 1: Cavity with Two-Coupled Ports

The cavity impedance can be written as:

$$Z_{cav} = \frac{R}{1 + i\,Q_o\left(\dfrac{\omega^2 - \omega_o^{\,2}}{\omega\omega_o}\right)} ,$$

where R is the equivalent shunt impedance of the cavity, Q_o, is the unloaded quality factor of the cavity, ω is the drive frequency, and ω_o is the resonant frequency of the cavity.

The coupling coefficients, β_1 and β_2 are defined[1] as,

$$\beta_1 = \frac{R}{N_1^{\,2} Z_{o1}} = \frac{Q_o}{Q_{ext1}} \text{ , and } \beta_2 = \frac{R}{N_2^{\,2} Z_{o2}} = \frac{Q_o}{Q_{ext2}} .$$

where Q_{ext1} and Q_{ext2} are the external quality factors of ports 1 and 2 respectively.

[1] This model is commonly used to describe a cavity coupled to a transmission line. A discussion of this model can be found in E.L. Ginzton, "Microwave Measurements", McGraw-Hill Book Co., 1957, Library of Congress Catalog Number 56-13393, pp. 391-397. For the development of the model see C.G. Montgomery, R.H. Dicke, E.M. Purcell, "Principles of Microwave Circuits", McGraw-Hill Book Co., 1948, Chapter 7.

Port 2 can be transformed into the cavity as shown in Fig. 2.

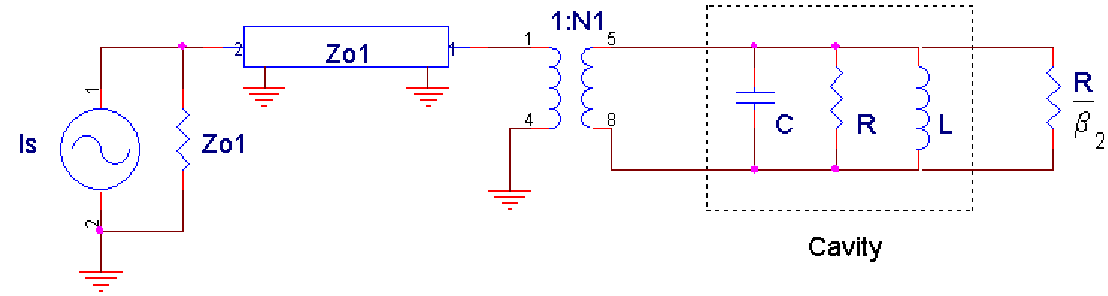

Figure 2: Cavity System showing Port 2 transformed into the Cavity

Similarly, the total impedance within the cavity can be transformed onto the input transmission line of port 1 as shown in Fig. 3.

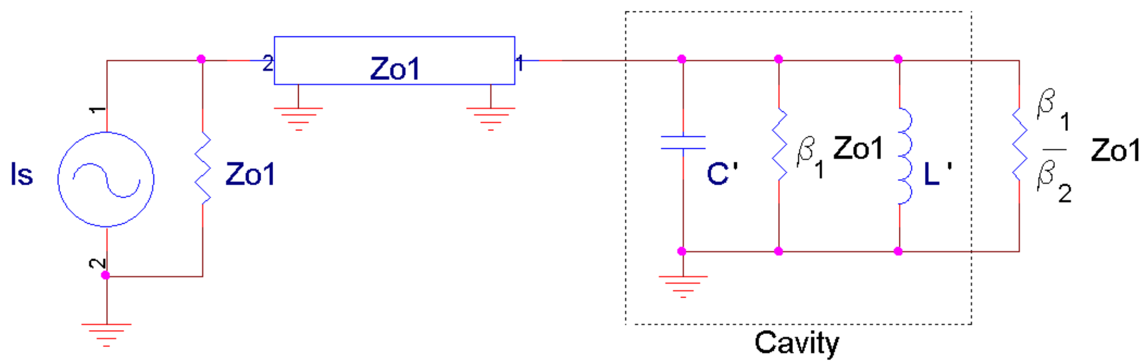

Figure 3: Cavity System showing all impedances reflected onto the Input T-Line

The impedance presented to the input transmission line can be expressed as

$$Z_L \quad = \quad \frac{\dfrac{\beta_1}{(1+\beta_2)} Z_{o1}}{1 \; + \; i \dfrac{Q_o \, Q_{ext2}}{Q_o + Q_{ext2}} \left(\dfrac{\omega^2 - \omega_o{}^2}{\omega \, \omega_o} \right)} \qquad .$$

On resonance ($\omega = \omega_o$), the load impedance presented to the transmission line is simply $\frac{\beta_1}{(1+\beta_2)} Z_{o1}$. Thus, the reflection coefficient[2], Γ_L, on resonance at the cavity end of the input transmission line can be expressed as

$$\Gamma_L = \frac{\beta_1 - \beta_2 - 1}{\beta_1 + \beta_2 + 1} \; .$$

In steady-state, the difference between the forward and reflected powers is equal to the power delivered to the cavity and to port 2. This is expressed as,

$$P_{FWD} \cdot \left(1 - \left|\Gamma_L\right|^2\right) = P_{cav} + P_{Zo2} \; ,$$

where P_{FWD} is the power associated with the forward wave on the input transmission line, P_{cav} is the power associated with cavity losses, and P_{Zo2} is the power delivered to the load in port 2. The power in the cavity and in the load of port 2 can be expressed in terms of the stored cavity energy, U_{cav}, and the quality factors Q_o and Q_{ext2} as follows,

$$P_{cav} = \frac{\omega_o U_{cav}}{Q_o} \quad \text{and} \quad P_{Zo2} = \frac{\omega_o U_{cav}}{Q_{ext2}} \; .$$

The sum of these two powers can be expressed as,

$$P_{cav} + P_{Zo2} = \frac{\omega_o U_{cav}}{Q_o}\left(1 + \beta_2\right) \; ,$$

where use of the relation $\beta_2 = \frac{Q_o}{Q_{ext2}}$ was used. Thus to maintain a certain steady-state cavity stored energy, the required forward power is

$$P_{FWD} = \frac{\left(1 + \beta_1 + \beta_2\right)^2}{4\beta_1} \cdot \frac{\omega_o U_{cav}}{Q_o} \; .$$

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[2] For a discussion of reflection coefficient and transmission line concepts see G. Gonzalez, “Microwave Transistor Amplifiers”, Prentice-Hall, Engelwood Cliffs, 1984, Chapter 1.